Effect of a possible cosmological time dependence of the gravitational parameter G on the peak luminosity of type Ia supernovae.

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February 2, 2008

Abstract

The cosmological expansion of the universe affects the behaviour of all physical systems and, in the case of gravitationally bound ones, could correspond to or mimic a time dependent Newton's constant. Here we discuss the case of a locally spherical mass distribution embedded in a generic Robertson Walker universe. Choosing the most appropriate metric tensor for the problem and assuming that the local time scale is much much lower than the cosmic one, we show that G is practically unaffected thus leaving the absolute magnitude of type Ia supernovae unaltered at all epochs.

1 Introduction

An open and since long time debated problem is the one of the effect of the cosmic expansion on the behavior of gravitating systems on a more or less local scale (a few examples out of the wide litterature are [1] [2] [3] [4] [5] [6]). Of the various ways this problem can affect the understanding of our universe, we would like to focus on the collapse of a white dwarf leading to a supernova of type Ia [7] [8] (SnIa). SnIa's are particularly important because they have led to the discovery of the accelerated expansion of the universe. The fundamental feature of SnIa's for the monitoring of the expansion is their being considered as "standard candles". The latter conviction is based on the fact that the peak luminosity of this kind of supernova is proportional to the mass of nickel synthesized during the collapse, which, to a good approximation, is a fixed fraction of the Chandrasekhar mass [9]. The Chandrasekhar mass, in its turn, is proportional to $G^{-3/2}$ [10] where G is the universal coupling constant between matter and geometry (Newton's constant).

There are alternative theories to General Relativity (GR) for which G is no constant at all, but rather it depends on cosmic time (examples are [11] [12] [13] [14] [15] [16] [17]). In that case, provided one has the explicit time dependence of "G", the absolute luminosity to be attributed to an SnIa should appropriately be corrected on the basis of the corresponding redshift z (distance in time) in order to separate the contribution of the expansion from the one of the changing G.

Even in the case of GR (possibly "extended" [18]), however, one could surmise an influence of the expansion of the universe appearing in the form of an effective G_{eff}, i.e. something "dressing" the universal G with a time- or z-dependence of the local gravitational interaction.

In any case the presence of a time (z) dependent G_{eff} (whatever be the origin of the dependence) would imply a correction on the absolute magnitude $M(z)$ of an SnIa, given by the formula [19]:

$$M(0) - M(z) = \frac{15}{4} \log \frac{G_{eff}(z)}{G_{eff}(0)}$$

Considering the relevance of this problem, in the present letter we shall focus on the supernova luminosity in any Robertson Walker (RW) spacely flat universe, i.e. any expanding universe in which the gravitational interaction is expressed by the geometry of a four dimensional manifold with a global symmetry reducing the number of independent functions, in the average cosmic metric tensor, to the only scale factor a.

We shall start from an ansatz for the metric describing the situation around a local spherical distribution of mass in an expanding RW universe. Then exploiting the fact that a supernova explosion is an extremely short event on the scale of cosmic times we shall verify that the implied G value is in any case the universal one.

2 A spherically symmetric perturbation in a Robertson Walker universe

Let us start from the remark that at the scale of a stellar system the effect of the cosmic curvature appears only as a very tiny perturbation of the usual stationary state solutions of general relativity. Considering for instance the example of a spherically distributed bunch of matter in a background RW universe with zero space curvature, we may guess and assume that locally the induced metric is essentially

$$ds^2 = (1 - f(r, \tau)) d\tau^2 - a^2(\tau) \left(\frac{dr^2}{1 - h(r, \tau)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \qquad (1)$$

where a is the cosmological scale factor (adimensional) and an extremely weak dependence of f and h on time is expected. Our τ is measured in meters.

Far away from the local source it must be $f,\, h \rightarrow 0$ so that the pure RW metric is recovered. On the other side if it were $a = a_0 =$ constant we could incorporate its value in the definition of r and from the field equations we would obtain the Schwarzschild solution $f = h = 2Gm/c^2 r = 2\mu/r$.

A metric like the one in (1) belongs to a general class of exact solutions of the Einstein equations discussed for instance in [20]. When applyed to a cosmological problem it has a partial correspondence with the early work of McVittie [1], many times reconsidered and discussed afterwards (see for instance [21] [22]). Actually, however, he analyzed a spherical massive source immersed in a homogeneous fluid with which it interacted gravitationally. Our case (and specific treatment) is different because for us the cosmic fluid is implicitly unaffected by the local spherical source, which assumtion is indeed realistic being the cosmic fluid itself the product of an average over the whole universe. What we want to elucidate is the inverse, i.e. the effect of the global (average) curvature on the local system.

Without loosing generality we may restyle (1) evidencing an arbitrary moment in cosmic time. The corresponding $a(\tau_0) = a_0$ (the value of the scale parameter at the chosen moment) may be absorbed into a rescaling of r, and we introduce the new renormalized scale factor $\alpha(\tau, \tau_0) = a(\tau)/a(\tau_0)$.

From the line element written in this form we can compute the Einstein tensor $G_{\mu\nu}$, assuming that $f,\, h << 1$ which is an obvious situation when black holes are not implied (actually, in "ordinary" conditions, $f,\, h <\sim 10^{-6}$):

$$\begin{aligned}
G_{00} &= 3\frac{\dot{\alpha}^2}{\alpha^2} - \frac{1}{r\alpha^2}\left(h' + \frac{h}{r}\right) - \frac{\dot{\alpha}}{\alpha}\dot{h} \\
G_{rr} &= \frac{1}{r}\left(f' + \frac{h}{r}\right) + \alpha\dot{\alpha}\dot{f} + \left(2\alpha\ddot{\alpha} + \dot{\alpha}^2\right)(f + h - 1) \\
G_{\theta\theta} &= \left(2\alpha\ddot{\alpha} + \dot{\alpha}^2\right) r^2 (f - 1) + \frac{1}{2}\alpha^2 r^2 \ddot{h} \qquad (2) \\
&\quad + \alpha\dot{\alpha} r^2\left(\dot{f} + \frac{3}{2}\dot{h}\right) + \frac{1}{2}r^2 f'' + \frac{1}{2}r\left(f' + h'\right) \\
G_{\phi\phi} &= G_{\theta\theta}\sin^2\theta \\
G_{0r} &= \frac{\dot{\alpha}}{\alpha}f' - \frac{\dot{h}}{r}
\end{aligned}$$

Primes mean partial derivatives with respect to r and dots correspond to partial τ derivatives.

One further assumption is that the source term in the Einstein equations can be decomposed into a cosmic contribution $T_{\mu\nu}$ (peculiar to the given model one is considering) and a local contribution $\mathfrak{T}_{\mu\nu}$, so that the equations can be written:

$$G_{\mu\nu} = \kappa T_{\mu\nu} + \kappa\mathfrak{T}_{\mu\nu}.$$

The two contributions are necessarily very different from one another and the second is locally much bigger than the first. Having assumed, on the average, a RW symmetry, we can verify, by direct inspection of (2), that the terms

appearing in the expression of $G_{\mu\nu}$ not containing f and h correspond to the cosmic source, so that the remaining ones pertain to the local source. If we consider the situation outside the local distribution of matter we are in "vacuo", so that the additional terms of (2) give rise to the equations

$$\begin{aligned}
\frac{1}{r\alpha^2}\left(h'+\frac{h}{r}\right)+\frac{\dot{\alpha}}{\alpha}\dot{h}=0 \\
\frac{1}{r}\left(f'+\frac{h}{r}\right)+\alpha\dot{\alpha}\dot{f}+\left(2\alpha\ddot{a}+\alpha^2\right)(f+h)=0 \\
\left(2\alpha\ddot{a}+\dot{\alpha}^2\right)r^2 f+\frac{1}{2}\alpha^2 r^2\ddot{h}+\alpha\dot{\alpha}r^2\left(\dot{f}+\frac{3}{2}\dot{h}\right) \\
+\frac{1}{2}r^2 f''+\frac{1}{2}r\left(f'+h'\right)=0 \\
\frac{\dot{\alpha}}{\alpha}f'-\frac{\dot{h}}{r}=0
\end{aligned} \tag{3}$$

The assumed weak dependence on time allows for a low order power series development around the local (in time) expression of the functions. Introducing $\mathfrak{t}=\tau-\tau_0$ as the time variable we write:

$$\begin{aligned}
\alpha(\mathfrak{t}) &= 1+H_0\mathfrak{t}-\frac{1}{2}q_0 H_0^2\mathfrak{t}^2+... \\
f(r,\mathfrak{t}) &= f_0(r)+f_1(r)\,\mathfrak{t}+f_2(r)\,\mathfrak{t}^2+... \\
h(r,\mathfrak{t}) &= h_0(r)+h_1(r)\,\mathfrak{t}+h_2(r)\,\mathfrak{t}^2+...
\end{aligned} \tag{4}$$

Just to fix the orders of magnitude let us remark that:

$$\begin{aligned}
f,h &\lesssim 10^{-6}\text{ at most} \\
H_0 &\sim 10^{-18}\text{ s}^{-1}\doteqdot\sim 10^{-26}\text{ m}^{-1}
\end{aligned}$$

The Hubble parameter correction can produce a contribution comparable to the one from f and h in times of the order of $\sim 10^5$ years.

Introducing (4) into (3) and fixing the attention on the zeroth order in $\mathfrak{t}$ we obtain:

$$\begin{aligned}
\frac{h_0'}{r}+\frac{h_0}{r^2}+H_0 h_1=0 \\
\frac{f_0'}{r}+\frac{h_0}{r^2}+H_0^2(1-2q_0)(f_0+h_0)+H_0 f_1=0 \\
h_2 r^2+H_0 r^2\left(f_1+\frac{3}{2}h_1\right)+\frac{r^2}{2}f_0''+H_0^2(1-2q_0)\,r^2 f_0 \\
+\frac{r}{2}\left(h_0'+f_0'\right)=0 \\
H_0 f_0'-\frac{h_1}{r}=0
\end{aligned} \tag{5}$$

The system (5) is easily solved, starting from the ansatz:

$$f_0 = 2\frac{\mu}{r}.$$

The solution is

$$\begin{aligned}
f_0 &= 2\frac{\mu}{r} \\
h_0 &= \frac{A}{r} + H_0^2 \mu r \\
f_1 &= -\frac{A-2\mu}{H_0 r^3} - H_0 \frac{\mu}{r} - H_0 \left(1-2q_0\right) \left(\frac{2\mu + A}{r} + H_0^2 \mu r \right) \\
h_1 &= -2H_0 \frac{\mu}{r} \\
h_2 &= \frac{3}{r^3}\left(\frac{A}{2} - \mu\right) + \frac{A}{r} H_0^2 \left(1-2q_0\right) + \frac{H_0^2}{r}\frac{7}{2}\mu + r\mu H_0^4 \left(1-2q_0\right)
\end{aligned} \tag{6}$$

where A is an integration constant.

In terms of the full functions we would write

$$\begin{aligned}
f\left(r,\mathfrak{t}\right) &= 2\frac{\mu}{r} - \left(\frac{A-2\mu}{H_0 r^3} + H_0 \frac{\mu}{r} + H_0 \left(1-2q_0\right) \left(\frac{2\mu + A}{r} + H_0^2 \mu r \right) \right) \mathfrak{t} + ... \\
h\left(r,\mathfrak{t}\right) &= \frac{A}{r} + H_0^2 \mu r - 2H_0 \frac{\mu}{r}\mathfrak{t} \\
&\quad + \left(\frac{3}{r^3}\left(\frac{A}{2} - \mu\right) + \frac{A}{r} H_0^2 \left(1-2q_0\right) + \frac{H_0^2}{r}\frac{7}{2}\mu + r\mu H_0^4 \left(1-2q_0\right) \right) \mathfrak{t}^2 + ...
\end{aligned}$$

In a flat, static background it would be $H_0 = 0$ and the solution should coincide with Schwarzschild's, so that it must be

$$A = 2\mu$$

and

$$f\left(r,\mathfrak{t}\right) = 2\frac{\mu}{r} - \mu \left(\frac{H_0}{r} + H_0 \left(1-2q_0\right) \left(\frac{4}{r} + H_0^2 r \right) \right) \mathfrak{t} + ...$$

$$\begin{aligned}
h\left(r,\mathfrak{t}\right) = \frac{2\mu}{r} + H_0^2 \mu r - 2H_0 \frac{\mu}{r}\mathfrak{t} \\
+\mu H_0^2 \left(r H_0^2 \left(1-2q_0\right) + \frac{11}{2r} - 4\frac{q_0}{r} \right) \mathfrak{t}^2 + ...
\end{aligned}$$

We see that the expansion of the universe shows up in a "decoupling" of h from f already at the zero order. However this correction is extremely small. In fact the zero order for h is:

$$\frac{2\mu}{r}\left(1 + \frac{H_0^2}{2} r^2\right)$$

and the difference from the typical Schwarzschild term appears (1% correction) at distances $\sim 10^{25}$ m $\doteqdot 10^3$ Mpc!

3 Behavior of test particles

In order to investigate further on the possible local effects of the expanding universe it is convenient to write down the geodesics for the metric in (1). Directly exploiting the spherical symmetry, which is not spoiled by the expansion, we may fix $\theta = \pi/2$, so that the three remaining independent equations for the motion of a test particle are

$$\frac{d^2\tau}{ds^2} + \alpha\dot{\alpha}\left(\frac{dr}{ds}\right)^2 + \alpha\dot{\alpha}r^2\left(\frac{d\phi}{ds}\right)^2 - \frac{\dot{f}}{2}\left(\frac{d\tau}{ds}\right)^2 + f'\frac{dr}{ds}\frac{d\tau}{ds} - \left(\frac{\dot{h}}{2}\alpha^2 + \alpha\dot{\alpha}\left(h+f\right)\right)\left(\frac{dr}{ds}\right)^2 - \alpha\dot{\alpha}fr^2\left(\frac{d\phi}{ds}\right)^2 = 0$$

$$\frac{d^2r}{ds^2} + 2\frac{\dot{\alpha}}{\alpha}\frac{dr}{ds}\frac{d\tau}{ds} - r\left(\frac{d\phi}{ds}\right)^2 - \frac{f'}{2\alpha^2}\left(\frac{d\tau}{ds}\right)^2 - \dot{h}\frac{dr}{ds}\frac{d\tau}{ds} - \frac{h'}{2}\left(\frac{dr}{ds}\right)^2 - hr\left(\frac{d\phi}{ds}\right)^2 = 0$$

$$\frac{d^2\phi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds} + 2\frac{\dot{\alpha}}{\alpha}\frac{d\phi}{ds}\frac{d\tau}{ds} = 0 \tag{7}$$

Eq. (7) is easily integrated to yield:

$$\frac{d\phi}{ds} = \frac{L}{\alpha^2 r^2} \tag{8}$$

being L a constant of the motion (angular momentum).

Using (8) the remaining two equations read:

$$\left\{\begin{array}{c} \frac{d^2\tau}{ds^2} + \alpha\dot{\alpha}\left(\frac{dr}{ds}\right)^2 + \alpha\dot{\alpha}r^2\left(1-f\right)\left(\frac{L}{\alpha^2r^2}\right)^2 \\ -\frac{\dot{f}}{2}\left(\frac{d\tau}{ds}\right)^2 + f'\frac{dr}{ds}\frac{d\tau}{ds} - \left(\frac{\dot{h}}{2}\alpha^2 + \alpha\dot{\alpha}\left(h+f\right)\right)\left(\frac{dr}{ds}\right)^2 = 0 \\ \frac{d^2r}{ds^2} + 2\frac{\dot{\alpha}}{\alpha}\frac{dr}{ds}\frac{d\tau}{ds} - r\left(1+h\right)\left(\frac{L}{\alpha^2r^2}\right)^2 \\ -\frac{f'}{2\alpha^2}\left(\frac{d\tau}{ds}\right)^2 - \dot{h}\frac{dr}{ds}\frac{d\tau}{ds} - \frac{h'}{2}\left(\frac{dr}{ds}\right)^2 = 0 \end{array}\right. \tag{9}$$

We may also directly obtain dr/ds from the line element (1):

$$\left(\frac{dr}{ds}\right)^2 = \frac{\left(1-f\right)\left(1-h\right)}{\alpha^2}\left(\frac{d\tau}{ds}\right)^2 - \frac{1-h}{\alpha^2} - \frac{L^2}{\alpha^4r^2}\left(1-h\right) \tag{10}$$

The last three equations are what is needed to pursue any further analysis of the behaviour of matter in our system.

3.1 A special case: a fixed object

We consider now a little object maintained at a fixed position $r = R$ in the field (by means of some appropriate force); the angular momentum will then be $L = 0$. In this case equations (9) become:

$$\begin{cases} \frac{d^2\tau}{ds^2} - \frac{\dot{f}}{2}\left(\frac{d\tau}{ds}\right)^2 = \mathfrak{g}_0 \\ \frac{f'|_{r=R}}{2\alpha^2}\left(\frac{d\tau}{ds}\right)^2 = -\mathfrak{g}_r \end{cases} \tag{11}$$

where $\mathfrak{g}$ is the four-vector representing the force per unit mass acting upon our object.

At this point let us introduce the approximate solutions for α, f, h and keep the 0 order in $\mathfrak{t}$:

$$\frac{d^2\tau}{ds^2} + H_0\mu\left(2qR + \frac{R}{2}H_0^2 + 8\frac{q}{H_0^2 R} + \frac{5}{2R}\right)\left(\frac{d\tau}{ds}\right)^2 = \mathfrak{g}_0$$

$$\frac{\mu}{R^2}\left(\frac{d\tau}{ds}\right)^2 = -\mathfrak{g}_r$$

The first equation tells us that, for $r = R$, $\frac{d\tau}{ds} = K(R)$ also is a constant (to say it better: it is a function of R).

This result is then introduced into the second equation, thus yielding (now $g(R) = c^2\frac{\mathfrak{g}(R)}{K^2(R)}$ and the explicit expression for μ has been restored):

$$-\frac{GM}{R^2} = g(R) \tag{12}$$

where g is in practice the local gravitational acceleration and of course (12) is Newton's law.

What is indeed remarkable is that the coupling constant is the universal G, unaffected by the expansion of the universe. A phenomenon which happens in a short time, such as the implosion of a star due to a gravitational collapse, will be determined and measured by the same value of G at all epochs. This is of paramount importance, as we noticed before, for the evaluation of the apparent luminosity of type Ia supernovae, which depends on $G^{-3/4}$.

4 Conclusion

We have studied the case of a spherical distribution of matter embedded in a RW spacely flat universe. Assuming that the global symmetry and behaviour of the universe are not affected by the local source, we have analyzed the influence of the cosmic expansion on the behaviour of the local system. What we have seen is that, for sufficiently small time spans (zeroth order expansion in time of the functions in the metric) the three-dimensional force acting upon a static test mass is indeed expressed by Newton's law without any epoch dependent renormalization of G. In this way the peak luminosity of SnIa's, which depends

on $G^{-3/2}$ and is due to a very rapid phenomenon, stays the same at all cosmic times. This result turns out to be valid for any RW type universe, i.e. for all theories where gravity is described by the curvature of a four-dimensional Riemannian manifold with isotropic homogeneous (in the mean) sources and flat three-dimensional space submanifold. This is true for instance for the Friedmann Robertson Walker, the Λ cold dark matter, and the cosmic defect theory [18]. In fact the result we have obtained could be anticipated considering that the gravitational interaction in an Einstein-type theory is always mediated by the unique coupling constant between matter and geometry, no matters where you are in cosmic time. Any cosmological effect shows up only on space and time scales far bigger than the ones of a local system.

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